

# Minimum Mass Solar Nebulæ and Planetary Migration

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## ABSTRACT

The Minimum Mass Solar Nebula (MMSN) is a protoplanetary disk that contains the minimum amount of solids necessary to build the planets of the Solar System. Assuming that the giant planets formed in the compact configuration they have at the beginning of the “Nice model”, Desch (2007) built a new MMSN. He finds a decretion disk, about ten times denser than the well-known Hayashi MMSN. The disk profile is almost stationary for about ten million years. However, a planet in a protoplanetary disk migrates. In a massive, long-lived disk, this question has to be addressed. With numerical simulations, we show that the four giant planets of the Solar System could not survive in this disk. In particular, Jupiter enters the type III, runaway regime, and falls into the Sun like a stone. Known planet-planet interaction mechanisms to prevent migration, fail in this nebula, in contrast to the Hayashi MMSN. Planetary migration constrains the construction of a MMSN. We show how this should be done self-consistently.

*Subject headings:* solar system: formation — accretion, accretion disks — methods: numerical

## 1. Introduction

The Minimum Mass Solar Nebula (hereafter MMSN) is the protoplanetary disk of solar composition that has the amount of metals necessary to build the eight planets of the Solar System (and the asteroid belts). From the masses and compositions of the planets, a density of solids is derived at several locations of the disk. Then, the solar composition is restored by adding gas, and a smooth protoplanetary disk density profile is derived. For a review of past works on the MMSN concept, see the introduction of Desch (2007). The most famous version of the MMSN was provided by Weidenschilling (1977) and Hayashi (1981). Assuming that the giant planets have a rocky-icy core of about 15 Earth masses, and that Jupiter accreted all the solid material available between 1.55 and 7 AU, Hayashi finds a surface density profile of  $\Sigma(r) = 1700(r/1 \text{ AU})^{-3/2} \text{ g.cm}^{-2}$ , similar to the one given by Weidenschilling (1977). Of course, the density profile obtained depends crucially on the position of the planets. Hayashi assumed that the planets formed where they presently orbit.

A recent model explains several features of the Solar System (the Late Heavy Bombardment, the orbital distribution of the Trojans of Jupiter, the orbital elements of the giant planets...) thanks to a late global instability in the outer Solar System dynamics (Gomes et al. 2005; Morbidelli et al. 2005; Tsiganis et al. 2005). In this so-called “Nice model”, the four giant planets were in a compact configuration just after the solar nebula dissipation. Therefore, if one assumes that this model is true, the Weidenschilling (1977) and Hayashi (1981) nebula is out of date. In a recent, very nice article, Desch (2007) constructed a new MMSN, assuming that the planets were formed in the disk at their starting position in the Nice model. He finds that the density distribution required to form the giant planets in this configuration, including the presence of an exterior disk of planetesimals, can be very well fitted by a steep power law density profile. In addition, he studies the time evolution of this disk. He finds a solution of a decretion disk, that can survive with an almost unchanged density profile in the giant planets region for a few million years. This enables the solid cores

of the four giants to reach their isolation masses, and then to slowly accrete their gaseous envelope. Consequently, this nebula is self consistent from the planetary formation point of view. The nice agreement between the initial position of the planets in the Nice model and a long-lived, almost power-law protoplanetary disk can be seen as a new plus point for the Nice model.

However, planets in gaseous disks are subject to planetary migration (see Papaloizou & Terquem 2006, for a review). How to prevent Jupiter (and Saturn) from becoming a hot Jupiter (or Saturn) is a longstanding problem in Solar System formation, but some solutions have been proposed. Masset & Snellgrove (2001) have shown that a mean motion resonance between Jupiter and Saturn can enable them to resist the driving inwards by the disk, and to migrate together outwards. Morbidelli & Crida (2007) found disk parameters for which Jupiter and Saturn enter the 3:2 mean motion resonance and then have a negligible migration. Morbidelli et al. (2007) then showed that Uranus and Neptune can also be saved in that case, by capture in resonance with Saturn. The four giant planets are in the end of the gas disk phase in a compact, fully resonant configuration. Two of these possible final configurations can lead to a late global instability of the dynamics, as required in the Nice model. These works were based on a disk in which the density slope was low and the gas density at the location of Jupiter was of the order of the one in the Hayashi (1981) nebula.

In this paper, we address the question of the planetary migration in the Desch (2007) nebula. In section 2, we describe our code, that enables a simulation of the entire disk, and of its global evolution. We also review the disk properties, and show how they should affect the planetary migration. The results are presented in section 3 for a locally isothermal disk, and in section 4 for a disk where the energy equation is taken into account. In every case, the planets are lost into the Sun. In section 5.1, we try to reproduce the Morbidelli & Crida (2007) result in the Desch (2007) disk, or in a colder disk with same density profile. In section 5.2, we focus on original results on Uranus and Neptune in some simulations. For comparison, the migration in the Hayashi (1981) MMSN is presented in section 6. Some numerical issues are discussed in section 7. Finally, our

conclusions are presented in section 8, and some prospectives are suggested. We show how planetary migration should be considered in the construction of a Minimum Mass Solar Nebula, and we draw some ideas towards a MMSN compatible with both planetary migration and the Nice model.

## 2. Numerical settings, disk properties and expected migration

### 2.1. Code description

We use the code FARGO (Masset 2000a,b), in its 2D1D version<sup>1</sup> (Crida et al. 2007). In this version, the classical 2D polar grid of the hydro-code (in which the planet-disk interactions are computed) is surrounded by a 1D grid. In the 1D grid, the disk is considered as axisymmetric, and only the radial component of the equations is computed ; this is computationally very cheap, therefore the 1D grid extension can be very large. The two grids are smoothly connected, so that the disk evolution is computed accurately over all the 1D grid extension : the evolution of the inner and outer parts of the disk computed in the 1D grid, influence the disk evolution in the 2D grid ; and reciprocally the planet-disk interactions computed in the 2D grid perturb the global disk evolution.

This property of the code makes it the perfect tool to study the behavior of the giant planets in the MMSN. Indeed, giant planets generally open gaps in the disk, and then are locked in the global viscous evolution of the disk (type II migration). Most often, this drives the planets inwards. But in the Desch nebula, the disk is viscously spreading. Consequently, its global evolution should drive planets in type II migration outwards. However, Crida et al. (2007) and Crida & Morbidelli (2007) found that things are not that simple. If the gap is not completely empty, a corotation torque is exerted on the planet, gas can pass from the inner to the outer disk (or reciprocally), and the planet can decouple from the disk evolution. Type II migration can then proceed inwards in a decretion disk, or outwards in a viscous accreting disk. This can be seen only with the use of the 2D1D algorithm, that permits both an accurate 2D computation of the planet-disk interaction and the 1D computa-

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<sup>1</sup>Now publicly available on <http://fargo.in2p3.fr>

tion of the global disk evolution. The global disk evolution also matters for the type III regime.

The outer edge of the 1D grid is located at 61 AU from the star, as prescribed by Desch (2007). The inner edge is located arbitrarily at 0.2 AU. The inner and outer edges of the 1D grid are open, allowing gas to flow out of the grid.

The rings of the grid are logarithmically distributed:  $\delta r/r$  is constant through the two grids. The cells of the 2D grid are squared:  $\delta r/r = \delta\theta$ .

The 2D grid extends from 1 or 1.6 to 23.5 AU, covering the planets region.

In the computation of the force of the disk on the planets, a part of their Hill sphere is excluded. To perform this, we calculate in the code the vector force  $\vec{F}_p$  using the following expression:

$$\vec{F}_p = \sum_{\text{cells}} \frac{GM_p \Sigma dS}{s'^3} f(s) \vec{s}, \quad (1)$$

where  $G$  is the gravitational constant,  $M_p$  the mass of the planet,  $dS$  is the surface of the considered cell,  $\vec{s}$  is the vector from the planet to the center of the cell,  $s$  is its length, and  $s' = \sqrt{s^2 + \epsilon^2}$  is the smoothed distance to the planet. The smoothing length  $\epsilon$  is  $0.6 r_H$ , where  $r_H$  is the Hill radius of the planet. The term  $f(s)$  is our filter used to exclude the neighborhood of the planet given by:

$$f(s) = \left[ \exp \left( -10 \left( \frac{s}{0.6 r_H} - 1 \right) \right) + 1 \right]^{-1}. \quad (2)$$

It is a smooth increasing function from 0 at  $s = 0$  to 1 when  $s \rightarrow \infty$ , through 0.1 when  $s = 0.47 r_H$ , 1/2 when  $s = 0.6 r_H$ , and 0.9 when  $s = 0.73 r_H$  (see Figure 2 of Crida et al. 2008).

## 2.2. Disk and planets parameters

The Solar Nebula found by Desch (2007) has the following properties. Assuming that Jupiter formed at 5.45 AU, Saturn at 8.18 AU, Uranus at 14.2 AU and Neptune at 11.5 AU, the gas density should be :

$$\begin{aligned} \Sigma_0(r) &= 343(r/10\text{AU})^{-2.168} \text{ g.cm}^{-2} \\ &= 5.68 \times 10^{-3}(r/1\text{AU})^{-2.168} M_\odot \text{AU}^{-2}. \end{aligned} \quad (3)$$

This is about 10 times more dense than the Hayashi (1981) nebula at 5 AU, and 6.4 times more dense at 10 AU, and 3 times more dense at 30 AU.

The steep density slope makes the disk a decretion disk, viscously spreading outwards, fed by the internal parts. The steady state profile found by Desch (2007) has a slightly different shape, but we use the power law for convenience. The two profiles are almost identical in the giant planets region. The outer edge of the disk is found to be located at 61 AU from the star.

The temperature is  $150(r/1\text{AU})^{-0.429} K$ , which corresponds to an aspect ratio of

$$(H/r)_0 = 0.05(r/1\text{AU})^{0.2855}. \quad (4)$$

The viscosity is given by an  $\alpha$  prescription (Shakura & Sunyaev 1973), with  $\alpha = 4 \times 10^{-4}$ .

With these characteristics, the disk profile remains almost unchanged for nearly 10 millions years, which leaves time for the solid cores of the planets to reach their isolation masses and accrete their atmosphere. However, this also leaves the planets time to migrate.

In our simulations, the planets are initially located on circular orbits at the position from which the Desch (2007) disk is derived (see above). The planets masses are grown smoothly from 0 to their present masses over 30 years at the beginning of the simulation. The planets are not accreting gas from the disk.

During the first 100 orbits of Jupiter (1274 years), the planets do not feel the disk potential, and therefore don't migrate. During this time, the planets launch a wake, open a gap, perturb the disk. This is for the disk to adapt to the planets potential and reach an equilibrium state. Then, the planets are released under the influence of the disk, and start their migration.

## 2.3. Expected migration

The aspect ratio is 8.1% at the location of Jupiter, and 9.1% at the location of Saturn. The Reynolds number  $\mathcal{R} = r^2 \Omega / \nu$  is  $3.8 \times 10^5$  at the location of Jupiter, and  $3.02 \times 10^5$  at the location of Saturn. Denoting  $\mathcal{P} = \frac{3}{4} \frac{H}{r_H} + \frac{50}{q\mathcal{R}}$ , we find  $\mathcal{P} = 0.876 + 0.131 \approx 1$  for Jupiter, and  $\mathcal{P} = 1.498 + 0.583 \approx 2$  for Saturn. The unified criterion to open a gap of depth 90% of the unperturbed density is  $\mathcal{P} \lesssim 1$  (Crida et al. 2006). Therefore, we expect Jupiter to open a non-empty gap, and Saturn not to perturb the density profile significantly.

Consequently the mechanism used by Morbidelli & Crida ( $r/r = 0.01$ , so that it is divided in 270 rings, and 628 sectors.

The mechanism used by Morbidelli & Crida (2007) to prevent Jupiter and Saturn migration should not work: it requires that the two planets lie in a wide common gap in mean motion resonance. On the contrary, Saturn should be here in type I migration, like Uranus and Neptune. Given that these three planets are massive (15 to almost 100 Earth masses), and that the disk density is high, their type I migration should be fast.

The isothermal horseshoe drag exerts on the planet a torque proportional to the logarithmic gradient of the vortensity:  $(d \ln(\Sigma/B)) / (d \ln r)$ , where  $B$  is the second Oort constant (Ward 1992; Masset 2001). In Keplerian rotation,  $B = \Omega/4$ . Thus, in the Desch (2007) nebula,  $\Sigma/B \propto r^{-0.668}$ , and  $d \ln(\Sigma/B)/d \ln r = -0.668$ . Due to the steep negative slope of the density profile, the corotation torque is negative, while it is zero if  $\Sigma \propto r^{-1.5}$  and positive if the slope is shallower. Such a disk density slope should therefore enhance the type I inwards migration of the planets with respect to more classical disks, in particular the Hayashi (1981) MMSN.

The high mass of the disk and the fact that Jupiter opens a small gap, makes it likely that Jupiter falls in the type III, runaway migration regime (Masset & Papaloizou 2003). Indeed, the mass of gas present in the coorbital region of Jupiter in the unperturbed disk is about 4 times the mass of Jupiter; therefore, the coorbital mass deficit may exceed the mass of the planet, which is the condition for the runaway migration.

In summary, in the Nebula found by Desch (2007), the four giant planets of the Solar System should migrate inwards on a short timescale. To check this, and to study the possible interactions between the four planets in the nebula, we have performed numerical simulations, presented in the next sections.

### 3. Models with locally isothermal equation of state

In this subsection, we use a locally isothermal equation of state  $P = c_s^2 \Sigma$ , with  $c_s$  the sound speed. The sound speed is a function of the distance to the star given by  $c_s^2 = (H/r)^2 GM_\odot/r \propto T$ , which is not evolving with time. The energy equation is not computed. The 2D grid ranges between 1.6 and 23.5 AU, with a resolution of

The planets are set according to section 2.2. The migration path obtained is displayed in Figure 1. As soon as released, Jupiter enters a type III migration regime and reaches 2 AU in 300 years. There, it stops due to interaction with the boundary of the 2D grid. If the grid had extended further inwards, there is no reason why Jupiter would not have reached the Sun.

Saturn also migrates inwards, at first in type I migration because it hardly perturbs the density profile. It enters the type III migration regime only at  $t = 3600$  years. Indeed, at this time, Saturn has reached 6.2 AU, where  $\mathcal{P}_{\text{Saturn}} = 1.88$ . Then, the dip dug by Saturn in the disk profile is deeper than before, the *coorbital mass deficit* can reach the mass of Saturn, and the runaway process can start.

When Saturn reaches 3.7 AU, at  $t = 3900$  years, it is caught in the 5:3 mean motion resonance with Jupiter: for 300 years, the ratio between their semi major axes is  $a_S/a_J = (5/3)^{2/3}$  (where  $a$  denote the semi major axis, and the subscript refers to the name of the planet). This breaks the runaway. At  $t = 4260$  years, Saturn has a close encounter with Jupiter and is kicked out. Then, it starts again an inwards migration until it is blocked again by Jupiter: from 5600 on, the ratio between their semi major axes is almost constant  $a_S/a_J \approx 1.43$ . This may be due to a mean motion resonance or to indirect interactions, Jupiter perturbing the disk and thus the motion of Saturn; in particular, the corotation torque should be positive and strong on the outer edge of the gap of Jupiter, which repels Saturn (see Masset et al. 2006a). The two planets then start to migrate slightly outwards in a common gap, like in Masset & Snellgrove (2001).

Neptune and Uranus are in the type I migration regime. Between 2600 and 3600 years, Neptune is slowed down by Saturn: the ratio of the semi-major axis of the two planets remains constant equal to  $a_N/a_S = 1.17 \approx (5/4)^{2/3}$  during this period. Between 3060 and 3520, Uranus is also slowed down, with  $a_U/a_S = 1.39 \approx (5/3)^{2/3}$ , and  $a_U/a_N \approx (4/3)^{2/3}$ . This shows likely captures of Neptune in the 5:4 resonance with Saturn, and Uranus in the 5:3 with Saturn or the 4:3 with Neptune. However, a mean motion resonance

can not be maintained if the involved bodies move away from each other. When Saturn accelerates its inwards migration, Neptune and Uranus stay behind.

Once Saturn has moved inward, the two ice giants accelerate again their inwards migration. Neptune goes faster and reaches an other resonance with Saturn. From 4800 years on, their semi-major axis ratio is constant equal to  $(7/5)^{2/3}$ .

At time 5900 years, Uranus reaches Neptune. The two planets then share the same average semi-major axis, equal to  $1.5^{2/3} a_S$ , which is characteristic of a 3:2 mean motion resonance with Saturn. Their configuration will be analyzed in more detail in section 5.2.

### 3.1. High resolution

Because some authors (D'Angelo et al. 2005) have found that type III migration is resolution dependent, we have performed the same experiments with a better resolution of  $\delta r/r = 10^{-2.5}$ . The 2D-grid is also extended inwards and ranges now from  $r = 1$  AU to  $r = 23.5$  AU. The 2D grid is now divided in 1000 rings and 1987 sectors. The 1D grid is divided in 1811 rings between 0.2 and 61 AU.

The result is displayed in Figure 2. It is very similar to the previous case. Jupiter still migrates in type III migration as soon as released, and Saturn also undergoes a runaway migration episode, but a few hundreds of years earlier than before. The planets go further inwards due to the extended 2D grid; this shows that their stop is a numerical artifact, as expected. The global result is not affected by the resolution. In fact, Figures 1 and 2 look similar, and the same explanations hold.

From 4100 years on, Saturn and Jupiter migrate slightly outwards in the 5:3 mean motion resonance, with  $a_S/a_J = (5/3)^{2/3}$ .

Neptune and Uranus have the same migration rate in both resolution cases, before 3000 years. After 3000 years, they migrate faster in the high resolution case because Saturn is not present anymore to slow them down. At  $t = 5400$ , a close encounter happens between Uranus and Neptune (the distance between the two planets is smaller than 0.4 AU). Then, the two planets share the same semi major axis again. More detail on their

configuration will be provided in section 5.2.

After  $10^4$  years, the four planets have escaped a fall into the Sun, thanks to the inner edge of the 2D grid. They have reached a fully resonant configuration in which their inwards migration is prevented. This is roughly similar to Morbidelli et al. (2007). However, here the four giant planets lie within 7 AU from the Sun, with Jupiter being at 3.6 AU. This is incompatible with the present structure of the main asteroid belt. In addition, the last planet is not stopped by a resonance with Saturn or with the last but one planet, so that the two ice giants share the same orbit. This is because the disk is more massive here than in Morbidelli et al. (2007), and the gaseous torques are stronger than the more distant resonances. It is very unlikely that this final configuration may lead to a late global instability that would drive the planets on their present orbits.

### 3.2. Getting rid of the hot Jupiter

In the previous simulations, the migration of Jupiter was stopped because the giant planet reached the inner edge of the 2D grid. Consequently, what happens after this event is admittedly interesting, but somehow artificial. In a new simulation, the planets are removed when they reach  $a < 2$  AU.

The results are displayed in Figure 3 for the higher of two resolutions used above ( $\delta r/r = 10^{-2.5}$ ). Jupiter being not present anymore, Saturn goes on migrating inwards and finally reaches  $a_S < 2$  AU as well. Then, Uranus and Neptune migrate inwards freely. The differential Lindblad torque, responsible for type I migration, is proportional to  $\Sigma r^4 \Omega^2 (H/r)^{-2}$ ; thus it increases as  $r^{-1.74}$  when the planets approach the Sun. This explains their accelerating migration. Within 8000 years, they have migrated inwards all the way to the grid edge too. The same happens at lower resolution.

With this more realistic way of dealing with the planets at the inner edge of the 2D grid, the four giant planets are lost in less than  $10^4$  years.

## 4. Non isothermal disk

It has been shown recently that computing the energy equation can change dramatically the corotation torque, and thus the migration rate of

low mass planets (Paardekooper & Mellema 2006; Baruteau & Masset 2008). Planets in type II migration should not be perturbed much, but this process may be critical for Neptune mass planets (Kley & Crida 2008). In the case considered in this paper, we expect this effect to be also significant for Saturn and Jupiter because their gap is not empty. To check this, we have implemented in FARGO-2D1D the computation of the Energy equation as done in the FARGO-ADSG version.

The Energy equation is the following, with  $e$  the surface density of internal energy of the gas,  $\vec{v}$  the velocity vector, and  $P$  the pressure:

$$\frac{\partial e}{\partial t} + \nabla(e\vec{v}) = -P\nabla\vec{v} + Q_+ + Q_- , \quad (5)$$

where  $Q_+$  and  $Q_-$  are heating and cooling terms, respectively.

The heating term comes from the viscous heating. The cooling term is given by a vertical black body emission:  $Q_- = 2\sigma_R T^4 / \kappa\Sigma$ , where  $\sigma_R$  is the Stefan-Boltzmann constant,  $T$  is the temperature, and  $\kappa$  is the opacity. The opacity is chosen such that the temperature profile given by Desch (2007) is an equilibrium profile of the unperturbed disk (in which  $Q_+ = 9\Sigma_0\nu\Omega^2/4$ ):

$$\kappa = \frac{8}{9} \frac{\sigma_R(H/r)_0^8}{\nu\Sigma^2 r} \quad (6)$$

with  $(H/r)_0$  given by Eq. (4).

The cooling time  $\tau_{\text{cool}} = e/Q_-$  is then  $(10/9\alpha)\Omega^{-1}$ ,  $\tau_{\text{cool}} = 2.78 \times 10^3 \Omega^{-1}$ . For Uranus and Neptune, the horseshoe libration time  $\tau_{\text{lib}} = 8\pi r_p/(3\Omega x_s)$  is  $326 \Omega^{-1}$  at 12 AU, with  $x_s = 1.16r_p\sqrt{q/(H/r)}$  (Masset et al. 2006b, Eq. (3)). The cooling time being large with respect to the libration time, the thermal effect on the corotation torque should saturate quickly and the migration of the ice giants should not proceed outwards like in Kley & Crida (2008).

Our equation of state is, with  $\gamma$  the adiabatic index ( $\gamma = 1.4$ ):

$$P = c_s^2 \Sigma / \gamma = (\gamma - 1)e . \quad (7)$$

Now, the sound speed  $c_s$  is not fixed but determined at every time-step by the local temperature, which is evolving with time.

The same experiments as in previous section are computed, with this energy equation and equation of state. The result at lower resolution is

shown in Figure 4. Jupiter and Saturn still undergo a type III migration episode. The migration of Uranus and Neptune is slower than with a locally isothermal disk, but is still directed inwards. In the end, the situation is basically unchanged with respect to previous section: Jupiter is blocked by the edge of the 2D grid, Saturn is blocked by Jupiter, Uranus and Neptune share a common orbit in resonance with Saturn.

With a higher resolution ( $10^{-2.5}$ ), and removing the planets when they reach the inner edge of the 2D grid, one gets the migration paths displayed in Figure 5. Jupiter and Saturn disappear below 1.3 AU within 4500 years. The migration of Uranus and Neptune is a bit slower than in the previous case. However, as soon as Saturn starts its type III inwards migration, the migration of the ice giants accelerates. Then the migration of Neptune is similar to the previous case (with a 800 years delay), while that of Uranus is much slower ( $\dot{a}_U \approx 2 \times 10^{-4} \text{ AU.year}^{-1}$ ). After 8800 years, Neptune reaches 1.1 AU and is declared lost. Uranus goes on migrating slowly inwards, accelerates after Neptune is suppressed, and reaches 1.3 AU after 15200 years.

## 5. Interactions between the planets

### 5.1. Jupiter and Saturn

Previous sections have shown that if the 4 giant planet of the Solar System are placed in the Desch (2007) nebula in the starting configuration of the Nice model and released simultaneously, they migrate inwards and disappear in the Sun in about  $10^4$  years. In this section, we try to prevent this by releasing the planets in sequence, and varying the disk aspect ratio. We aim at reproducing the best candidate mechanism so far to prevent the inwards migration of Jupiter and Saturn: Masset & Snellgrove (2001) and Morbidelli & Crida (2007) have shown that when Jupiter and Saturn orbit in a mean motion resonance in a common gap, they can avoid type II migration or even migrate outwards.

Therefore, we have performed simulations in which Jupiter is held on a fixed circular orbit longer than Saturn and the ice giants. The three lighter planets migrate inwards. Once Saturn's semi major axis has reached a constant value in terms of Jupiter semi major axis (which betrays

a resonance), we release Jupiter as well. Immediately, Jupiter migrates inwards in type III migration and disappears; and the three other planets migrate inwards again. This is because the considered disk is too thick to enable Jupiter to open a deep and wide gap, embracing also Saturn.

So, we decrease the aspect ratio to  $H/r = 0.04$ . As the viscosity is given by an  $\alpha$ -prescription, the viscosity is also smaller. In fact,  $\mathcal{P}_{\text{Jupiter}} = 0.467$ . The planets are maintained on a fixed orbit for 5250 years (500 Jupiter orbits). At this time, Jupiter has opened a deep and wide gap, the density at the bottom of which is a bit less than 2% of the unperturbed density. Saturn has opened a partial gap, of depth a half of the unperturbed density, and the two gaps have merged (see the solid profile in Figure 6, compared to the dot-dashed initial profile). This kind of gap suits the Masset & Snellgrove (2001) mechanism: Jupiter and Saturn lie together between the inner and the outer disk.

First, only Saturn is released. One expects that Saturn, repelled by the outer disk, would migrate inwards until it encounters a mean motion resonance with Jupiter. However, Saturn goes immediately in type III migration outwards and reaches 10 AU. Twenty orbits later, Jupiter, Uranus and Neptune are released as well. Jupiter runs away inwards and reaches 2 AU in about a hundred years. If the 4 planets are released simultaneously after 500 orbits of Jupiter, the same thing happens.

In an other attempt, Uranus and Neptune are not considered, and the aspect ratio is set constant equal to 0.05, like in the almost stationary solution found by Morbidelli & Crida (2007). Saturn is placed on a circular orbit at 10 AU, and let free to migrate after 2550 years (200 Jupiter orbits), while Jupiter is still held on a fixed orbit at 5.45 AU. Saturn migrates inwards in type III migration, and then is blocked by Jupiter. Then, we release Jupiter as well after 5100 years (400 Jupiter orbits). Again, Jupiter runs in type III migration into the Sun as soon as released.

Decreasing even more the viscosity and aspect ratio of the disk may help, but  $H/r = 0.04$  and  $\alpha = 4 \times 10^{-4}$  are already rather small compared to the standard values inferred from observations. In fact, there are two reasons why the Masset & Snellgrove (2001) mechanism can not work in the considered disk, even if one assumes

that it is cool enough to enable Jupiter to open a wide gap :

1. The density of this disk is so high that Jupiter (or Saturn) can easily enter the runaway type III migration regime. In that case, Jupiter is driven inwards faster than Saturn, and the putative resonance is broken.
2. This disk is a decretion disk. The Masset & Snellgrove (2001) mechanism prevents a pair of planets from inwards type II migration. It works when the innermost planet is more massive than the outer one, and repels the latter outwards against the accreting outer disk. Here, the global evolution of the disk is directed outwards. A more massive Saturn than Jupiter would be needed to prevent the outwards type II migration of the two planets.

## 5.2. Uranus and Neptune

In the above simulations, when the migration of Jupiter is artificially stopped at the edge of the 2D grid, Uranus and Neptune always end on the same orbit, captured in the same mean motion resonance with Saturn. This strange configuration is worth a more detailed analysis.

In the first case presented (locally isothermal disk, with resolution  $\delta r/r = 10^{-2}$ ), the distance between the two planets – shown as crosses in Figure 7 – is bounded by 0.1 AU after  $t = 6000$  years. The Hill radius of the planets is  $r_H = a_p(M_p/3)^{1/3} = 0.14$  AU at  $a_p = 5.5$  AU. This indicates a satellite motion of the planets. This kind of configuration requires a lot of damping because the planets were of course unbound gravitationally at the beginning. The damping is here provided by the disk, and is visible between 5900 and 6000 years in Figure 7: the maximum distance between the planets decreases until it becomes smaller than the Hill radius. The high density of this nebula makes such a satellite capture possible.

In the second case (locally isothermal,  $\delta r/r = 10^{-2.5}$ ), the planets are not satellites of each other. Here, the distance between Uranus and Neptune is slightly oscillating around 2 AU after 6000 years. They share the same orbit, with an eccentricity smaller than 0.04 and Uranus leading about  $30^\circ$  before Neptune: the angle between Uranus

and Neptune, seen from the Sun, is plotted as crosses in Figure 2, with respect to the right  $y$ -axis. The two planets are not at a Lagrange point of the other one. This unusual configuration may be due to the mean motion resonance with Saturn, or more probably to the strong dissipation by the disk, that modifies the local dynamics. Once the disk will have evaporated, these two planets should have a tadpole motion around one of their  $L_4/L_5$  Lagrange points. Such a configuration have already been studied, for instance by Thommes (2005); Beauge et al. (2007); Cresswell & Nelson (2008). Generally, it is assumed that a giant planet captures a migrating terrestrial planet, that may then grow. In contrast, here the effect of the giant planet is to put two 15 Earth mass planets together on the same orbit. This is a new way of forming Trojan planets.

In the third case (with energy equation, and  $\delta r/r = 10^{-2}$ ), the distance between the two ice giants (shown as the + symbols in Figure 4) settles to about 2 times their semi-major axis. This indicates that the two planets are in opposition, librating around their  $L_3$  point. Once again, this configuration is not possible in the absence of gas dissipation or of the other planets, because the  $L_3$  point is unstable. If the other planets and the disk disappear, these two planets should have coorbital horseshoe orbits.

These three original outcomes open new possibilities for extra-solar systems. They do not fit the Solar System configuration, though. Whether or not a scenario like the Nice model may apply from such 1:1 resonances, is not clear. In particular, the configurations with Uranus and Neptune at opposition or at 30 degrees on the same orbit are probably unstable on the short term in absence of the gas disk. This is thus unlikely to yield to a *late* instability, as required. In the case where the two planets are satellite of each other at a distance  $d = 0.1$  AU, the energy required to separate them,  $(GM_{\text{U}}M_{\text{N}}/d - GM_{\text{U}}M_{\text{N}}/r_H)/2$ , is roughly equivalent to the energy required to eject a Moon size body out of the Solar System from a 10 AU orbit. Therefore, it is likely that they stay in this satellite configuration.

## 6. Migration in the Weidenschilling (1977) - Hayashi (1981) nebula

For comparison, the same experiment as in section 3 has been performed in the standard Minimum Mass Solar Nebula. The initial density is the one quoted in introduction, that is:

$$\Sigma(r) = 1.9125 \times 10^{-4} (r/1 \text{ AU})^{-3/2} M_{\odot} \cdot \text{AU}^{-2}. \quad (8)$$

This profile appears in Figure 6 as the straight, gray, dotted line.

The equation of state is locally isothermal. The aspect ratio is given by Eq. (4), and the viscosity is given by an  $\alpha$ -prescription with  $\alpha = 4 \times 10^{-4}$ . The planets are initially placed on circular orbits at their present position:  $a_J = 5.2$  AU,  $a_S = 9.6$  AU,  $a_U = 19.2$  AU,  $a_N = 30$  AU. The resolution is  $10^{-2}$ , the 2D grid extends radially from 2.15 to 52 AU, while the 1D grid covers the range [0.312 ; 104] AU.

The planets do not feel the disk potential during 400 Jupiter orbits (4740 years), and are then released. The evolution of the semi major axes of Jupiter and Saturn are displayed in Figure 8. In spite of the fact that the shape of the gaps opened by the planets are similar to the ones in the Desch (2007) disc (the width and depth depend only on  $M_p$ ,  $H/r$  and  $\nu$ , see Crida et al. 2006), no type III migration take place, because this disc is not massive enough. Saturn migrates faster than Jupiter ; about 2600 years after the release, the two planets enter in 2:1 mean motion resonance, which slows Saturn down. After 25000 years, the migration stops at  $a_J = 4.75$ ,  $a_S = 7.57$  AU.

In an other, longer term simulation, we change the aspect ratio to  $H/r = 0.05$  and the viscosity to  $\alpha = 4 \times 10^{-3}$ . The low aspect ratio enables Jupiter to open a significant gap, and Saturn a shallow one. The density profile after 400 Jupiter orbits can be seen as the bottom, gray, dashed line in Figure 6. The migration paths are displayed in Figure 9 for  $10^5$  years. Jupiter migrates inwards in type II migration, while Saturn migrates faster. After 6400 years, Saturn encounters the 2:1 mean motion resonance with Jupiter, but is not caught in it. At about  $10^4$  years, Saturn encounters the 5:3 mean motion resonance with Jupiter ; from this time on,  $a_S/a_J \approx (5/3)^{2/3}$ . The gaps of the two planets merge. Then, the Masset & Snellgrove (2001) mechanism

takes place, and the two planets have very little migration, like in Morbidelli & Crida (2007) : their semi major axes don't change by more than 1 AU in  $10^5$  years.

Note that the stop of the migration of Jupiter at 4.5 (or 4.75) AU is not due to the edge of the 2D grid : the radius of the inner edge of the 2D grid is about a half of the semi major axis of Jupiter. The presence of Saturn is responsible for the salvation of Jupiter. To check this, the same simulation has been performed with Jupiter alone, without the other three planets. The migration path is displayed in Figure 9 as the orange dashed line. In this case, Jupiter has an unperturbed type II migration, and reaches 3 AU in 40 000 years, 2.7 AU in 50 000 years.

For comparison, the migration path of Jupiter in Figure 5 is displayed also in Figure 9 as a dotted red line. It looks like a free fall towards the Sun.

Uranus and Neptune are migrating inwards in type I migration. Their migration speed is slower than in the previous cases, because the density is moderate and the density slope is such that the isothermal corotation torque is zero. When approaching Saturn, Uranus slows down and almost stops between 9 and 10 AU. Then, Neptune is caught in the 2:3 mean motion resonance with Uranus at  $8.5 \times 10^4$  years : from this date on, their semi-major axis ratio is  $1.5^{2/3}$ .

## 6.1. Conclusion

Migration also occurs in the Hayashi (1981) nebula, and it is not possible that the giant planets stay where they form until the present time. Therefore, the Hayashi (1981) nebula is not more consistent with planetary migration than the Desch (2007) one. However, no runaway migration takes place, and the Jupiter-Saturn pair can avoid migration all the way to the Sun for some reasonable disk parameters. Then, a scenario like the one presented in Morbidelli et al. (2007) is possible. In this article, the authors find 6 fully resonant configurations of the four giant planets in which the migration is prevented. After disk dissipation, two of these configurations can lead to a late global instability of the planetary system, compatible with the Nice model.

## 7. Discussion

### 7.1. Energy Equation and cooling law

To slow down migration in the Desch (2007) nebula, one can think of taking the energy equation into account with an other opacity law. This would change the thermal structure of the disk. With a shorter cooling time, the temperature and the aspect ratio decrease. This has basically two effects: (i) Saturn and Jupiter would open a deeper gap, but our locally isothermal simulations with  $H/r \leq 0.05$  show that it may not be enough to prevent their migration ; (ii) the thermal horseshoe effect would not saturate if  $\tau_{\text{cool}} \sim \tau_{\text{lib}}$ , which could lead to a positive corotation torque on the planets, particularly Uranus and Neptune.

This should be the subject of a future, dedicated study. However, it is rather unlikely that the four planets stay exactly on their initial orbit until the disk dissipates, in order for the Nice model to take place.

### 7.2. Accretion

In all the simulations presented so far, the planets were not allowed to accrete. This may appear unrealistic at first sight, but on the other hand, accretion must have stopped anyhow before the masses of the planets exceed the present masses. This is certainly a critical issue for the formation of Jupiter and Saturn, because the accretion timescale for so massive planets should be very short. In both density profiles studied here, free accretion of the planets leads to too high masses. This open problem requires a complete, dedicated study, that is beyond the scope of this paper. We make the assumption that some mechanism prevented accretion to go on when Jupiter and Saturn reached their actual masses. However, a few comments are in order.

In the simulation presented in Figure 9, the gas density at the bottom of the gap created by Saturn is of the order of 3 times the one at the bottom of the gap of Jupiter. If this situation lasts a few hundred thousand years, Saturn may accrete more gas than Jupiter on the long term. This would be a problem for the Masset & Snellgrove (2001) mechanism : a lighter outer planet than the inner one is required for the migration to be slowed down, stopped, or reversed. In the Solar

System though, we know that the mass of Saturn somehow remained smaller than a third of the mass of Jupiter. This mass ratio allows the migration of the pair to be negligible over the disc life time, with suitable disk parameters. Therefore, the Masset & Snellgrove (2001) mechanism applies.

An other issue with accretion concerns type III migration. D'Angelo & Lubow (2008) have observed that allowing the planet to accrete, may prevent the runaway migration. However, this is most likely because their accreting planets grow too fast from low mass planets in type I migration to high mass planets in type II migration in a clean gap. With accretion turned on, but the planet mass artificially kept constant to  $3 \times 10^{-4} M_*$ , they find that type III migration occurs, which shows that accretion should not perturb too much the corotation torques responsible for the runaway. To check this, we have restarted the simulation presented in section 3.1 and Figure 2 at the time where the planets are released, turning accretion by Jupiter and Saturn on. Accretion is computed using the recipe by Kley (1999), with a timescale to accrete all the gas within  $0.45r_H$  of the planet equal to 1.6 year, that is 1/8 Jupiter orbit. The evolution of the mass and semi major axis of Jupiter are displayed in Figure 10 as the curves with + symbols, compared to the solid line taken from Figure 2. Type III migration is confirmed (and even enhanced), even if Jupiter reaches 5 Jupiter masses in a century.

For all these reasons, and even if a deeper study of the accretion processes would certainly be valuable, we think that our results are robust as far as this issue is concerned.

## 8. Summary and conclusion

In this paper, we have confronted the Desch (2007) Solar Nebula to the planetary migration issue. This disk has a high gas density, a high density slope, and this profile remains almost unchanged for 10 million years. Therefore, the question of the planetary migration should not be eluded. The 2D1D version of FARGO is particularly suited to such a study. The physical parameters of the disk are fixed, but a broad range of numerical parameters have been studied (resolution, energy equation, fate of the planets that

reach the inner edge of the 2D grid), as well as the influence of the aspect ratio.

The high density slope gives a negative non thermal horseshoe torque, that accelerates even more the inwards type I migration of Uranus and Neptune. The high density permits type III migration of massive planets. In particular, we can not avoid an extremely fast inwards migration of Jupiter. In spite of several attempts, it seems to be impossible to prevent a loss of all the giant planets in less than  $2 \times 10^4$  years, which is 2 to 3 orders of magnitude shorter than the disk life time. Taking into account the energy equation with a cooling compatible with the assumed temperature structure of the disk does not change the result.

Preventing the migration of the giant planets requires interactions between the planets. However, as this disk is a massive, decretion disk, the Masset & Snellgrove (2001) mechanism to prevent Jupiter and Saturn from migrating inwards can not be applied. Shortly said, we conclude that the Desch (2007) nebula is incompatible with our present knowledge of planetary migration.

So, we are facing a problem. Either this new MMSN makes the planetary migration in the Solar System an even more critical issue and makes obsolete the former solutions to prevent migration of the giant planets (Morbidelli & Crida 2007; Morbidelli et al. 2007), or the existence of planetary migration makes this MMSN questionable (in particular the apparently unavoidable type III migration of Jupiter).

A possible solution has been laid out in section 6. If one believes that the Nice model is true, it is clear that the outer planets should have ended the disk phase in a more compact configuration than the present one. However, the migration of the planets can account for a formation on a larger radial range, followed by a compaction of the configuration of the giant planets, like in Figure 9. Then, a fully resonant configuration can be achieved, that can be compatible with the Nice model, like in Morbidelli et al. (2007). In addition, the solid material that built the giant planets may come not only from the region around their respective orbits: dust drifts inwards in a protoplanetary disk (Weidenschilling 1977), and small bodies migrate as well, so that the giant planets region may be replenished in solids by the outer parts of the disk. This enables the formation of

Jupiter, Saturn, Uranus and Neptune in a less massive disk than the Desch nebula, to avoid the type III migration of Jupiter. In section 6, we have seen that this is possible in the old Hayashi (1981) nebula. However, there is no reason why the giant planets should have formed exactly where they presently orbit; in particular the presence of an outer cold disk of planetesimals (required in the Nice model) is problematic if Neptune formed beyond 25 AU. Thus, a new construction of a MMSN is needed, that takes into account the Nice model and the planetary formation constraints, like in Desch (2007), and also the migration of planets and planetesimals.

In any case, our results show that planetary migration should be considered in the construction of a MMSN. The location where the planets form determines the gas density profile of the nebula, which determines the migration path of the planets, which drives the planets to a new position after the disk dissipation. This final configuration, and not the initial one, should be compatible with the Nice model (or with the present configuration if one does not believe that the Nice model is true). This idea requires a detailed study, that is beyond the scope of this paper, but the results presented here advocate for such a self-consistent construction of the Minimum Mass Solar Nebula.

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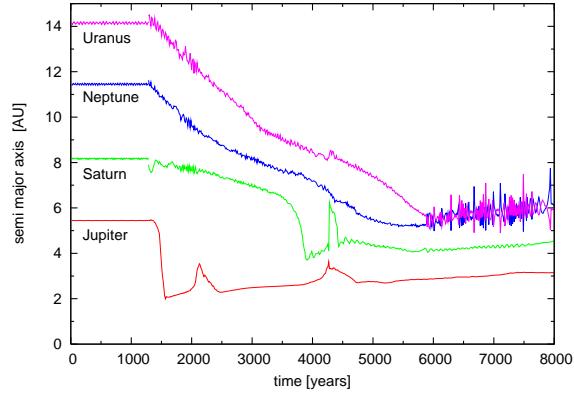


Fig. 1.— Migration path of Jupiter, Saturn, Uranus and Neptune in the Desch (2007) nebula, with locally isothermal EOS and resolution of  $10^{-2}$ .

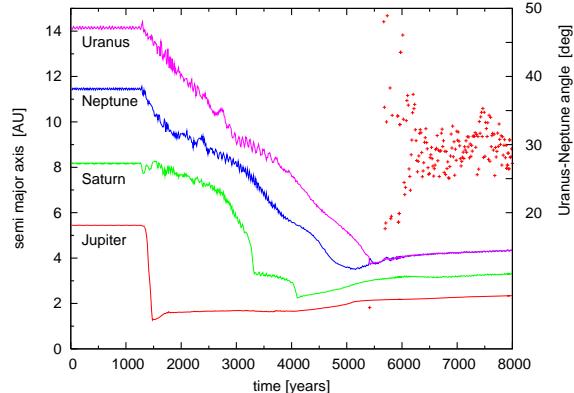


Fig. 2.— Migration path of Jupiter, Saturn, Uranus and Neptune in the Desch (2007) nebula, with locally isothermal EOS and resolution of  $10^{-2.5}$  (curves, left  $y$  axis). The crosses show the angle between Uranus and Neptune when they share the same orbit (right  $y$ -axis).

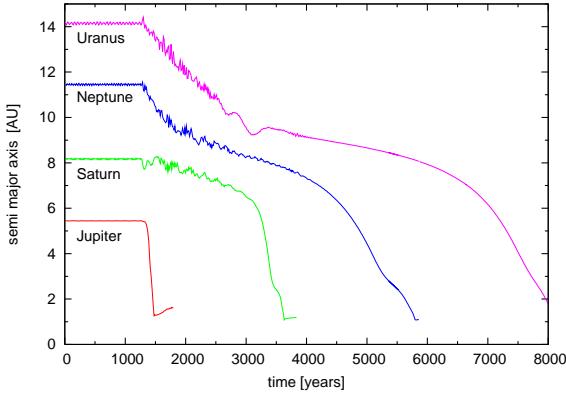


Fig. 3.— Migration path of Jupiter, Saturn, Uranus and Neptune in the Desch (2007) nebula, with locally isothermal EOS and resolution of  $10^{-2.5}$ . The planets are removed from the simulation when they reach the 2D-grid inner edge.

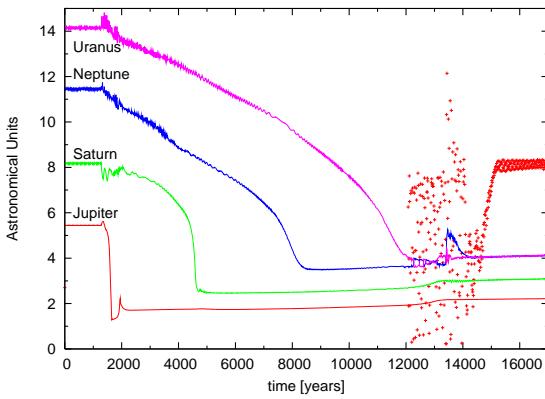


Fig. 4.— Migration path of Jupiter, Saturn, Uranus and Neptune in the Desch (2007) nebula, with Energy equation computed, and resolution of  $10^{-2}$  (curves). Crosses: Uranus-Neptune distance.

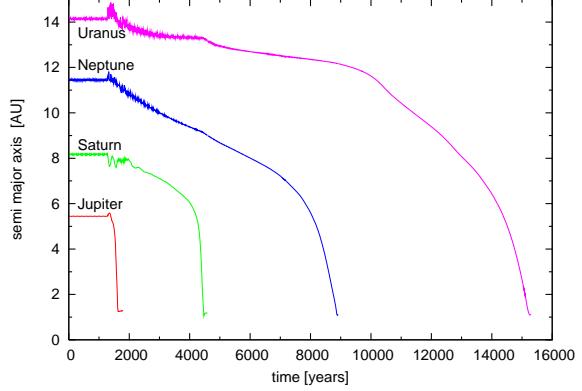


Fig. 5.— Migration path of Jupiter, Saturn, Uranus and Neptune in the Desch (2007) nebula, with Energy equation computed, and resolution of  $10^{-2.5}$ . The planets are removed when they encounter the inner edge of the 2D grid.

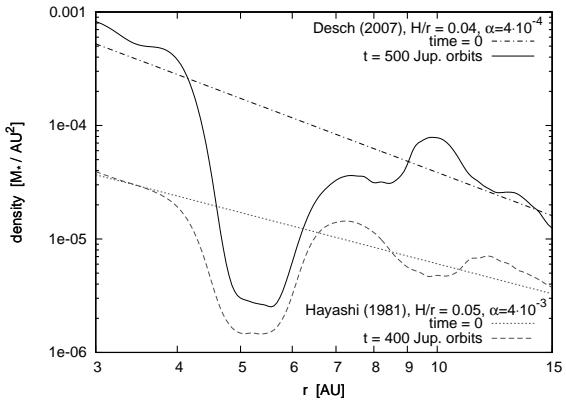


Fig. 6.— Top black lines: Initial density profile (dot-dashed, straight line) and profile after 500 Jupiter orbits (solid curve) in the Desch (2007) nebula with  $H/r = 0.04$  (see section 5.1). Bottom gray lines: Initial density profile (dotted, straight line) and profile after 400 Jupiter orbits (dashed curve) in the Hayashi (1981) nebula (see section 6).

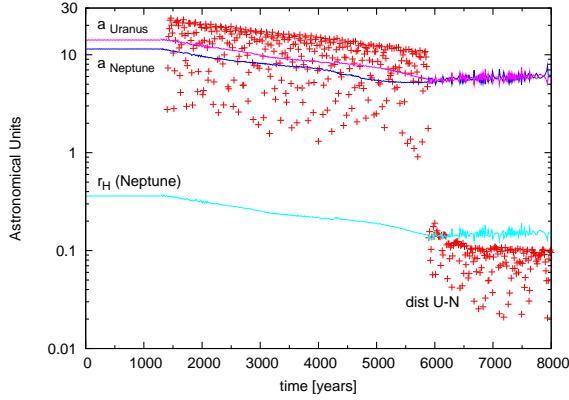


Fig. 7.— Migration path of Uranus and Neptune taken from Figure 1 (two top curves labeled “ $a_{\text{Uranus}}$ ” and “ $a_{\text{Neptune}}$ ” respectively). The size of Neptune Hill radius is the bottom light blue curve (“ $r_H(\text{Neptune})$ ”). The distance between the two planets is shown with + symbols, labeled “dist U-N”.

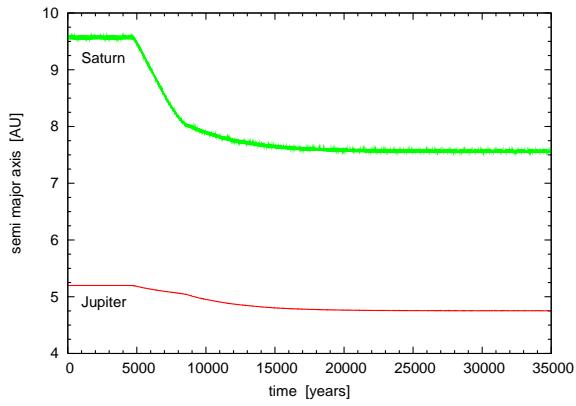


Fig. 8.— Migration path of Jupiter (bottom red curve) and Saturn (top green curve) in a disk with the Hayashi (1981) density profile,  $H/r = 0.05(r/1\text{AU})^{0.2855}$  and  $\alpha = 4 \times 10^{-4}$  like in the Desch (2007) nebula. The Equation of State is locally isothermal. The resolution is  $\delta r/r = 10^{-2}$ .

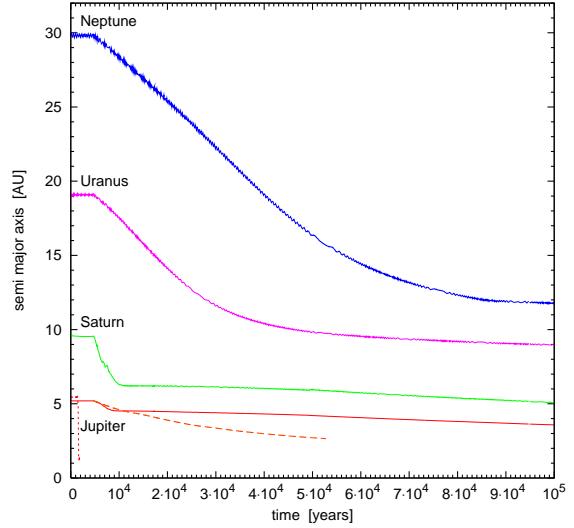


Fig. 9.— Migration path of Jupiter, Saturn, Uranus and Neptune in a disk with the Hayashi (1981) density profile,  $H/r = 0.05$ ,  $\alpha = 4 \times 10^{-3}$ . The Equation of State is locally isothermal. The resolution is  $\delta r/r = 10^{-2}$ . The orange dashed line corresponds to a simulation where Jupiter alone is migrating in the same disk. The red dotted curve is taken from Figure 5, for comparison.

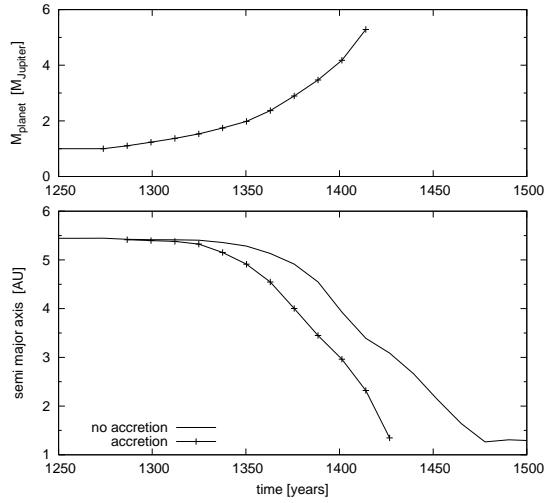


Fig. 10.— Bottom panel: Migration path of Jupiter in the Desch (2007) disk, with locally isothermal equation of state and resolution  $10^{-2.5}$  without accretion (solid line, taken from Figure 2), or with accretion with timescale 1.6 year (solid line with + symbols). Top panel: evolution of mass of the planet in the accreting case.